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Frequency and Magnitudes of Monthly and Annual Flow Rates (As Determined By (a) A Normal Distribution (b) By Ranking (c) A Gamma Distribution And (d) By A Log Normal Distribution)

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FREQUENCY AND MAGNITUDES OF MONTHLY AND ANNUAL FLOW RATES

As Determined By (a) A Normal Distribution (b) By Ranking (c) A Gamma Distribution And (d) By A Log Normal Distribution

A Computer Program of Generalized Use

by

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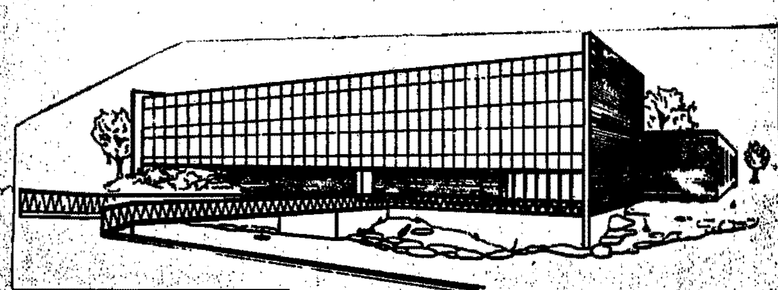

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BACKGROUND OF PROGRAM

In developing a water resources atlas for the State of Utah it was necessary to determine the characteristics of stream flows within the State. The variations of stream flow for any given month from year to year as well as the variations from month to month throughout the year are important characteristics which were determined by analyzing the streamflow records to determine their probability distributions. The program which is described in the following pages was written to determine the monthly as well as annual runoff amounts which might be expected for any number of specified levels of probability.

Since all streamflow data does not follow the same distribution function, the program has been designed to fit the data to any or all of the following four distribution functions which are commonly used. 1. The normal distribution; 2. The distribution actually obtained from the data by ranking it from high to low; 3. The gamma distribution; and 4. The long-normal distribution.

The program has been written specifically to determine the monthly and annual runoff volumes in ac-ft and also in inches over the watershed. The program could, however, serve equally well for determining monthly and annual amounts of other quantities at specified levels of probabilities after very minor modifications.

PROBABILITY ANALYSES

Normal probability

The common bell shaped normal probability curve has the equation

$$y = \frac{1}{\sigma\sqrt{2\pi}} \; e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < \infty \quad . \; . \; (1)$$

Under the assumption that the data follows a normal distribution, an estimate, $\bar{x}$, of the mean μ is the average of the observed data, and an estimate, s, of the standard deviation, σ, and the coefficient of skewness, g, are given respectively by

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \bar{x} \sum_{i=1}^{n} x}{n-1}} \quad . \; . \; . \; . \; . \; . \; . \; (2)$$

$$g = \frac{n \sum_{i=1}^{n} (x_i - \bar{x})^3}{(n-1)(n-2)\, s^3} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (3)$$

where n is the number of observations.

The probability that the value of x will be equal to or less than a given specified amount is the area under the density function (1) to the left of that value of x and equals the integral of (1) from $-\infty$ to the value. Since (1) requires a numerical integration, and because small data samples are used to estimate the parameters, μ and σ, values of x at specified levels of probable occurrence are commonly determined by

$$x = \bar{x} \pm st \quad . \quad (4)$$

where the values of t are obtained from a table for the specified value of probability and the value of $n-1$ (degrees of freedom). The t table is used because the statistic, $t_o = \frac{x - \bar{x}}{s}$, calculated from a sample of size n from a normal distribution follows a t distribution with $n-1$ degrees of freedom.

<u>Ranked distribution</u>

By listing data in order of its magnitude (either from high to low or from low to high) a data series or frequency series is obtained. By assuming that the record is representative of future events, the probability, p, of any event being equaled or exceeded, in this frequency series can be obtained by

$$p = \frac{m}{n+1} \quad . \quad (5)$$

where m is the rank number, and n is the total number of data points in the series. The curve obtained by plotting the probability against the magnitude of the event is the accumulated distribution curve of the actual data. The derivative would represent the actual density function of the data just as (1) represents the density function assuming a normal distribution.

Gamma distribution

The gamma distribution involves two parameters r and λ and the density function is defined by (see Parzen[1]).

$$f(x) = \frac{\lambda}{\Gamma(r)} (\lambda x)^{r-1} e^{-\lambda x} \qquad \begin{array}{c} 0 < x < \infty \\ r > 0 \\ \lambda > 0 \end{array} \quad \ldots\ldots\ldots \quad (6)$$

The parameters λ and r can be estimated by noting that

$$\mu = \frac{r}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$\sigma^2 = \frac{r}{\lambda^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

giving the estimating equations $x = \bar{x}/s^2$ and $r = \bar{x}^2/s^2$. The gamma function $\Gamma(r)$ can be obtained by several relationships and/or from tables. In general, however, $\Gamma(r)$ can be calculated from the product power expansion

$$\Gamma(r) = \frac{1}{r} \prod_{k=1}^{\infty} \frac{\left(1 + \frac{1}{k}\right)^r}{\left(1 + \frac{r}{k}\right)} \quad \ldots\ldots\ldots\ldots \quad (9)$$

Because of the slow converges of this product series, a numerical procedure by Davis[2] has been used in the program to compute $\Gamma(r)$ for r less than 2. For values of r greater than or equal to 2, $\Gamma(r)$ is obtained by $\Gamma(r) = (r-1)(r-2)(r-3) -- (r-T_r)\,\Gamma(T_r)$ where

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[1] Parzen, E.: Modern Probability Theory and Its Applications, Wiley, 1960.

[2] Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, U. S. Dept. of Commerce, Natl. Bur. of Standards, Applied Mathematics Series 55, U. S. Government Printing Office, Washington, D. C. See page 256, June 1964.

$T_r = r - [r-1]$, $[r-1]$ being the truncated value of the quantity, and $\Gamma(T_r)$ is obtained by the numerical procedure.

The gamma distribution is more versatile than the normal distribution and in many respects is more realistic for events such as rainfall and runoff, that have only positive values. Consequently, among others the gamma distribution (and also the incomplete gamma function, see Barger and Thom[3]) have found wide application in predicting rainfall probabilities.

Log-normal distribution

The log-normal distribution is obtained by the transformation $y = \ln x$ and by assuming that the transformed data follow a normal distribution. Consequently, the procedures for determining a log-normal distribution are first to take the logarithm of the original data and thereafter follow the procedures used to fit a normal distribution to the data. Finally, the expected magnitudes at the specified probabilities are obtained as the antilogarithms. If logarithms to the base e are used, this last step can be accomplished by

$$x = e^y \qquad (10)$$

where y is the magnitude obtained by eq. (4) using the transformed data.

[3] Barger, G. L. and H. C. S. Thom: "Evaluation of Drought Hazard," Agronomy Journal, Vol. 41, 1949.

FORTRAN IV PROGRAM

The program is designed to determine the expected runoff for each month in the year as well as the annual for the specified levels of probability, by any or all of the four distributions methods discussed earlier. The program assumes that the annual as well as the twelve monthly values are contained on the data cards. By a slight modification to the program the annual runoff could be computed.

After computing and printing the pertinent parameters for the distribution, the program prints the values of the runoff at each of the specified probability levels in units of ac-ft and also in terms of inches over the watershed contributing to the measured flow (see page 18 for sample of output). A listing of the FORTRAN IV program along with a description of the variable names used in the program is given beginning on page 11.

INPUT DATA REQUIRED BY PROGRAM

The proper order of the data cards required by the program are shown in figure 1. The first data card contains the number of probability levels and the percentage levels for which the runoff values are to be determined. (If the normal or log-normal distributions are called for, the number must be odd and the probability levels symmetric about the 50 percent level.) The next group of data cards contains the values from a t-table. The number of these cards equals the truncated number obtained by dividing the number of probability levels by two. Each of these cards in consecutive order contains the t-value for the specified probability levels working from the extremes toward the 50 percent level for 5, 10, 15, 20, 25, 30 and 60 degrees of freedom respectively. The next data card contains the FORMAT which is to be used to read the monthly and annual values of runoff, and the following card contains the name of the stream gaging station. The final card preceding the data is the control card. The number of the basin is in columns 1 and 2 (i.e. the first two digits of the USGS number). The station number is in columns 7 through 10 (i.e. the last four digits of the USGS number). The last two digits of the beginning year of record are in columns 14 and 15, and the final year of record in columns 19 and 20. In columns 21 through 30 the area (in square miles) of the watershed should be punched with the decimal point in column 25. The values punched in columns 35, 40, 45, 50, and 55 respectively determine whether the original data is listed, whether a normal distribution, ranking, a gamma distribution and/or a log normal distribution are to be used. If the distribution analysis is not wanted a zero should be given to the respective control parameter. If several stations of data are to be processed during the same access to the computer only the last two of these cards should be placed before the data for subsequent stations. A 99 in columns 4 and 5 on the two final cards terminates execution.

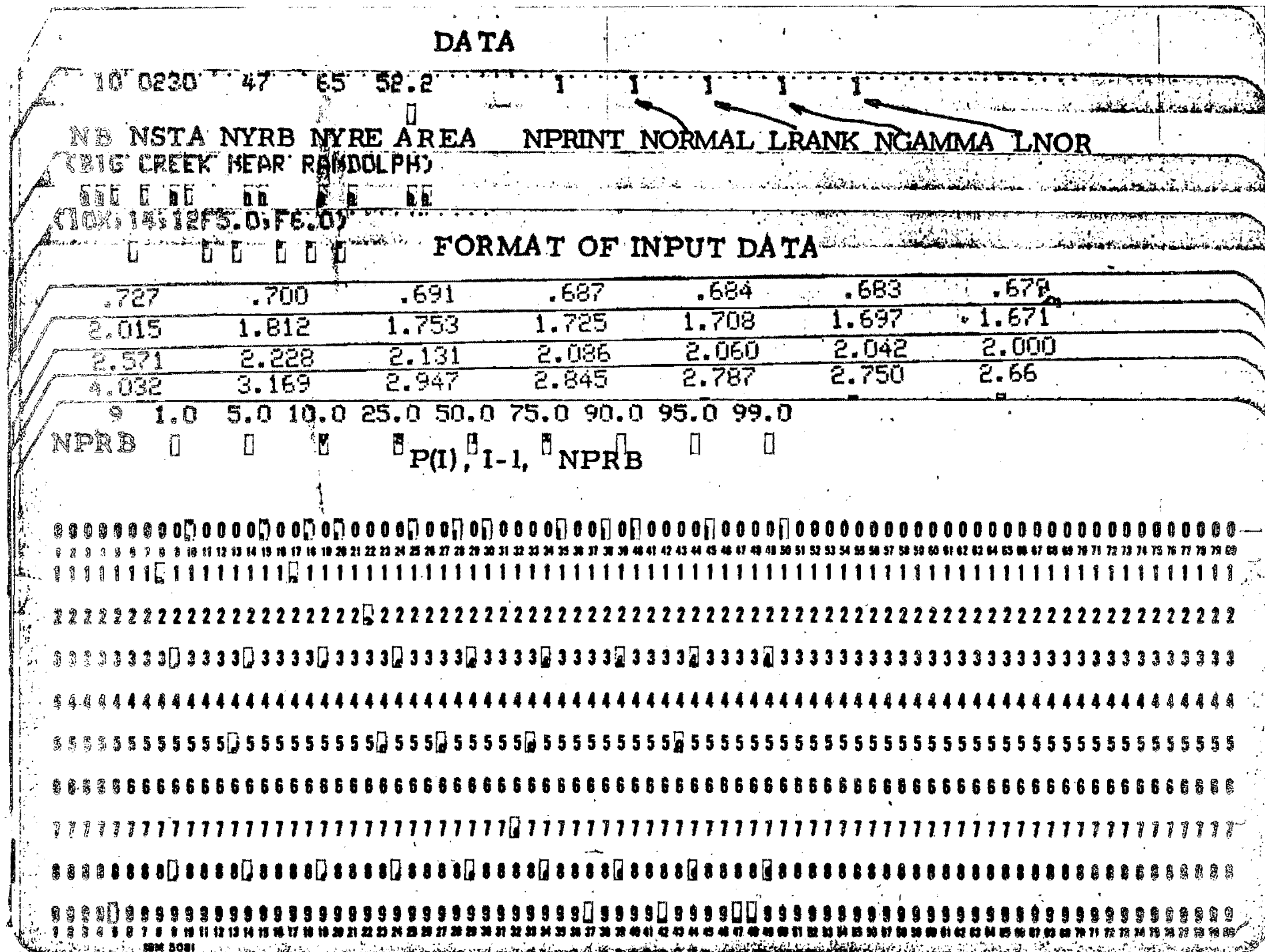

Figure 1. Sample of input control cards.

LIST OF VARIABLE NAMES

CK	an array of numbers supplied by a data statement which are used to numerically approximate the gamma function.
RPIE	parameter used in computing gamma function.
GAMMA1	parameter used in computing gamma function.
FMT	an array which stores the FORMAT which is read in at execution time.
NAME	an array used to store the name of the station being processed.
R	a two dimensional array used to store the monthly run data.
AV	an array used to store the average values of runoff for each month.
STD	an array used to store the standard deviation of runoff for each month.
SKEW	an array used to store the coefficient of skewness of runoff for each month.
P	an array containing the desired levels of probability.
T	a two dimensional containing the values from a t-table.
RUN	a two dimensional array used to store data for convenience in output.
RUN1	an array used to store data for convenience in output.
PLEV	an array used to store the probability level of ranked data, and also intermediate probability level from the gamma distribution.
XR	a two dimensional array used to store runoff values for convenience in output.
FXR	a two dimensional array used to store density function values for convenience in output.
AMBA	an array used to store the values of the λ parameter of the gamma distribution.

GAMMA	an array used to store the values of the gamma function.
RPAR	an array used to store the values of r parameter of the gamma distribution.
NPRB	the number of probability levels.
NPH	the number of probability levels on each side of 50 percent.
NB	the basin number.
NSTA	the station number.
NYRB	the beginning year of the data.
NYRE	the ending year of the data.
AREA	the area of the watershed drainage area in square miles.
NPRINT	Controls output--if equal to zero original data is not printed.
LNOR	if equal to zero log normal distribution analysis is not performed.
NGAMMA	if equal to zero gamma distribution analysis is not performed.
NRANK	if equal to zero rank distribution analysis is not performed.
NORMAL	if equal to zero normal distribution analysis is not performed.
NN & FN	number of years of record.
RLANGR	a function subroutine used for Lagrange's interpolation or extrapolation based on a second degree polynomial.

Note: The additional variables used in the program used for convenience in computation and are defined in the FORTRAN statements in which they appear.

```
      C  C
      &IN FOR MONF
            DOUBLE PRECISION CK(26),RPIE,GAMMA1,AMBAX
            REAL FMT(12),NAME(12)
            REAL R(67,13),AV(13),STD(13),SKEW(13),P(11),T(5,7),RUN(13,5),
           $RUNI(13),PLEV(67),XR(10,13),FXR(10,13),AMBA(13),GAMMA(13),RPAR(13)
      COMPUTES THE DATA NEEDED TOP PLOT THE BAR GRAPHS OF MONTHLY RUNOFF
            DATA (CK(I),I=1,26)/1.0,.5772156649015329,-.6558780715202538,
           $-.0420026350340952,.1665386113822915,-.0421977345555443,
           $-.0096219715278770,.0072189432466663,-.0011651675918591,-.000215241
           $6741149,.0001280502823882,-.0000201348547807,-.0000012504934821,
           $1.1330272320E-6,-2.056338417 E-7,6.116095 E-9,5.0020075 E-9,
           $-1.1812746 E-9,1.043427 E-10,7.7823 E-12,-3.6968 E-12,5.1 E-13,
           $-2.06 E-14,-5.4 E-15,1.4 E-15,1.0 E-16/
            READ(5,110) NPRB,(P(I),I=1,NPRB)
            NPH=NPRB/2
            NPH1=NPH+1
            DO 10 I=1,NPH
         10 READ(5,127)(T(I,J),J=1,7)
        127 FORMAT(7F10.5)
        110 FORMAT(I5,11F5.1)
            READ(5,100) (FMT(I),I=1,12)
        100 FORMAT(12A6)
         50 READ(5,100)(NAME(I),I=1,12)
            READ(5,101) NB,NSTA,NYRB,NYRE,AREA,NPRINT,NORMAL,LRANK,NGAMMA,LNOR
        101 FORMAT(4I5,F10.2,5I5)
            IF(NB .EQ. 99) GO TO 99
            CONV=.01875/AREA
            NYRBM=NYRB-1
            NN=NYRE-NYRB+1
            NN5=NN/5
            FN=NN
            FN1=FN-1.0
            FN2=FN-2.0
            DO 4 J=1,13
          4 AV(J)=0.0
            IF(NPRINT .EQ. 0) GO TO 2
            WRITE(6,103) NB,NSTA,( NAME(I),I=1,12)
            WRITE(6,104)
        104 FORMAT(1H ,129HYEAR     OCT.      NOV.      DEC.      JAN.      FEB.
           $  MARCH     APRIL     MAY       JUNE      JULY      AUG.      SE
           $PT.   ANNUAL)
          2 DO 1 I=1,NN
            READ(5,FMT) II,(R(I,J),J=1,13)
            IF(NPRINT .GT. 0) WRITE(6,102) II,(R(I,J),J=1,13)
            DO 1 J=1,13
          1 AV(J)=AV(J)+R(I,J)
        103 FORMAT(1H1,25HORIGINAL DATA FOR STATION,I3,1H-I4,12A6)
        102 FORMAT(1H ,I4,     4F9.0,9F10.0)
            IF(NORMAL .EQ. 0) GO TO 48
            WRITE(6,153)
        153 FORMAT(1H1,30HBASED ON A NORMAL DISTRIBUTION)
            DO 5 J=1,13
          5 AV(J)=AV(J)/FN
            WRITE(6,132) NB,NSTA,(NAME(I),I=1,12)
        132 FORMAT(1H0,44HAV. STD. DEV. AND COEF. OF SKEW. FOR STATION,I3,1H-,
           $I4,12A6)
```

```
      WRITE(6,104)
      WRITE(6,124) (AV(J),J=1,13)
  124 FORMAT(1H ,4HAVE.,4F9.0,9F10.0)
      DO 6 J=1,13
      SX2=0.0
      SX3=0.0
      AVE=AV(J)
      DO 7 I=1,NN
      SX=R(I,J)-AVE
      SXX=SX*SX
      SX2=SX2+SXX
    7 SX3=SX3+SX*SXX
      VAR=SX2/FN1
      STD(J)=SQRT(VAR)
    6 SKEW(J)=FN*SX3/(FN1*FN2*VAR*STD(J))
      WRITE(6,105) (STD(J),J=1,13)
  105 FORMAT(1H ,4HSTD.,4F9.0,9F10.0)
      WRITE(6,106) (SKEW(J),J=1,13)
  106 FORMAT(1H ,4HSKEW,4F9.2,9F10.2)
      WRITE(6,108) NB,NSTA
  108 FORMAT(1H0,65HTABLE OF RUNOFF VALUES AT INDICATED PROBABILITY LEVE
     $L FOR STATION,I3,1H-,I4)
      WRITE(6,109)
  109 FORMAT(1H ,129HPROB LEVEL OCT.  NOV.      DEC.      JAN.      FEB.
     $    MARCH     APRIL     MAY       JUNE      JULY       AUG.     SEPT
     $.      ANNUAL)
      IF(NN5 .GT. 0) GO TO 9
      WRITE(6,107) NN
  107 FORMAT(1H ,I5,56HLESS THAN 5 YEARS OF RECORD SO LEVELS ARE NOT DET
     $ERMINED)
      GO TO 11
    9 IF(NN5 .GT. 7) NN5=7
      DO 8 I=1,NPH
      FAC=T(I,NN5)
      DO 12 J=1,13
      FACD=FAC*STD(J)
      RUN1(J)=AV(J)+FACD
   12 RUN(J,I)=AV(J)-FACD
      WRITE(6,111) P(I),(RUN1(J),J=1,13)
      DO 23 J=1,13
   23 RUN1(J)=CONV*RUN1(J)
    8 WRITE(6,112) (RUN1(J),J=1,13)
  112 FORMAT(1H ,4H IN.,4F9.3,9F10.3)
  111 FORMAT(1H ,F4.1,4F9.0,9F10.0)
      WRITE(6,111) P(NPH1),(AV(J),J=1,13)
      DO 51 J=1,13
   51 RUN1(J)=CONV*AV(J)
      WRITE(6,112) (RUN1(J),J=1,13)
      DO 13 I=1,NPH
      IM=NPH1-I
      II=NPH1+I
      WRITE(6,111) P(II),(RUN(J,IM),J=1,13)
      DO 24 J=1,13
   24 RUN(J,IM)=CONV*RUN(J,IM)
   13 WRITE(6,112) (RUN(J,IM),J=1,13)
   48 IF(LRANK .EQ. 0) GO TO 47
   11 DO 22 J=1,13
```

```
      M=NN
   14 M=M/2
      IF(M) 22,22,16
   16 K=NN-M
      JJ=1
   17 I=JJ
   18 L=I+M
      IF(R(L,J)-R(I,J)) 21,21,20
   20 B=R(I,J)
      R(I,J)=R(L,J)
      R(L,J)=B
      I=I-M
      IF(I-1) 21,18,18
   21 JJ=JJ+1
      IF(JJ-K) 17,17,14
   22 CONTINU
      WRITE(6,154)
  154 FORMAT IH0, / 31H PROBABILITIES BASED ON RANKING)
      WRITE(6,113) NB,NSTA,(NAME(I),I=1,12)
  113 FORMAT(IH0,35HRANKED VALUES OF RUNOFF FOR STATION,I3,1H-,I4,12A6)
      WRITE(6,109)
      FNP=100./(FN+1.0)
      DO 25 I=1,NN
      PLEV(I)=FNP*FLOAT(I)
   25 WRIT (6,142) PLEV(I),(R(I,J),J=1,13)
  142 FORMAT(1H ,F5.1,5F9.0,8F10.0)
      WRITE(6,108) NB,NSTA
      WRITE(6,109)
      J=2
      NN1=NN-1
      DO 26 I=1,NPRB
      PO=P(I)
   27 PLJV=.5*(PLEV(J)+PLEV(J+1))
      IF(PO .LT. PLJV .OR. J .EQ. NN1) GO TO 28
      J=J+1
      GO TO 27
   28 P1=PLEV(J-1)
      P2=PLEV(J)
      P3=PLEV(J+1)
      DO 29 K=1,13
   29 RUN1(K)=RLANGR(PO,P1,P2,P3,R(J-1,K),R(J,K),R(J+1,K))
      WRITE(6,111) PO,(RUN1(K),K=1,13)
      DO 30 K=1,13
   30 RUN1(K)=CONV*RUN1(K)
   26 WRITE(6,112)  (RUN1(K),K=1,13)
   47 IF(NGAMMA .EQ. 0) GO TO 247
      WRITE(6,486) NB,NSTA,(NAME(I),I=1,12)
  486 FORMAT(IH0,/9H STATION ,I2,1H-,I4,16H NAME OF STATION,12A6)
      WRITE(6,421)
  421 FORMAT(IH0,82HRUNOFF IN AC-FT AND INCHES FOR GIVEN PROBABILITY LEV
     $ELS USING A GAMMA DISTRIBUTION)
      WRITE(6,420)  (P(I),I=1,NPRB)
  420 FORMAT(1H ,5HPROB.,F10.2,4F12.2,6F11.2)
      WRITE(6,422)
  422 FORMAT(1H ,5HMONTH)
      DO 400 J=1,13
      IF(LRANK .EQ. 0) GO TO 401
```

```
      RMINI=R(NN,J)
      RMAXI=R(1,J)
      GO TO 402
  401 RMINI=R(1,J)
      RMAXI=R(1,J)
      DO 403 I=2,NN
      IF(R(I,J) .GT. RMAXI) RMAXI=R(I,J)
  403 IF(R(I,J) .LT. RMINI) RMINI=R(I,J)
  402 IF(NORMAL .GT. 0) GO TO 404
      AV(J)=R(1,J)
      SX2=AV(J)*AV(J)
      DO 405 I=2,NN
      AV(J)=AV(J)+R(I,J)
  405 SX2=SX2+R(I,J)*R(I,J)
      AVE=AV(J)/FN
      VAR=(SX2-AVE*AV(J))/(FN-1.0)
      AV(J)=AVE
      GO TO 410
  404 VAR=STD(J)*STD(J)
  410 RINIC=RMINI-2.*(AV(J)-RMINI)/FN
      IF(RINIC .LT. 0.0) RINIC=0.0
      AVE=AV(J)-RINIC
      AMBA(J)=AVE/VAR
      RPAR(J)=AVE*AMBA(J)
      RBEGG=RMINI-RINIC
      IF(RPAR(J) .GT. 1.0) GO TO 473
      RPAR(J)=1.0
      AMBA(J)=1.0/AVE
  473 RPAR1=RPAR(J)-1.0
      NRDIF=RPAR(J)-1.
      RRPR=RPAR(J)-FLOAT(NRDIF)
      RPIE=1.
      GAMMA1=0.0
      DO 415 I=1,26
      RPIE=RPIE*RRPR
  415 GAMMA1=GAMMA1+CK(I)*RPIE
      GAMMA1=1.0/GAMMA1
      IF(NRDIF .EQ. 0) GO TO 407
      DO 416 I=1,NRDIF
  416 GAMMA1=GAMMA1*(FLOAT(NRDIF-I)+RRPR)
  407 GAMMA(J)=GAMMA1
      RPIE=AMBA(J)
      GAMMA1=DLOG(RPIE/GAMMA1)
      RDIF=(RMAXI-RMINI)/9.
C 407 RDIF=(RMAXI-RMINI)/9.
      DO 408 I=1,10
      XR(I,J)=RBEGG+FLOAT(I-1)*RDIF
      AMBAX=RPIE*XR(I,J)
      RPE=GAMMA1+RPAR1*DLOG(AMBAX)-AMBAX
  408 FXR(I,J)=EXP(RPE)
      PLEV(1)=0.0
      RDIF1=54.166667*RDIF
      RDIF2=4.166667*RDIF
      XR1=XR(1,J)*XR(1,J)
      XR2=XR(2,J)*XR(2,J)
      DEN=XR1*XR(2,J)-XR(1,J)*XR2
      ACOEF=(FXR(1,J)*XR(2,J)-FXR(2,J)*XR(1,J))/DEN
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```
      BCOEF=(XR1*FXR(2,J)-XR2*FXR(1,J))/DEN
      PLEV(2)=33.333333*ACOEF*XR1*XR(1,J)+50.*BCOEF*XR1
      DO 412 I=2,9
  412 PLEV(I+1)=PLEV(I)+RDIF1*(FXR(I-1,J) +FXR(I,J))-RDIF2*(FXR(I-2,J)
     $+FXR(I+1,J))
      PLEV(11)=PLEV(10)+50.*RDIF*(FXR(9,J)+FXR(10,J))
      IF(PLEV(11) .LT. 100.0) GO TO 464
      RATIP=99.9999/PLEV(11)
      DO 413 I=2,11
  413 PLEV(I)=RATIP*PLEV(I)
  464 XRRP=0.0
      K=2
      DO 423 I=1,NPRB
      PO=P(I)
  427 PLJV=.5*(PLEV(K)+PLEV(K+1))
      IF(PO .LT. PLJV .OR. K .EQ. 10) GO TO 423
      K=K+1
      XRRP=XR(K-2,J)
      GO TO 427
  423 RUN1(I)=RLANGR(PO,PLEV(K-1),PLEV(K),PLEV(K+1),XRRP,XR(K-1,J),XR(K,
     $J))+RINIC
      WRITE(6,430) J,(RUN1(I),I=1,NPRB)
  430 FORMAT(1H ,I5,5F12.0,6F11.0)
      DO 424 I=1,NPRB
  424 RUN1(I)=CONV*RUN1(I)
  400 WRITE(6,431) (RUN1(I),I=1,NPRB)
  431 FORMAT(1H ,5X,5F12.3,6F11.3)
      WRITE(6,433)
  433 FORMAT(1H0,33HPARAMETERS FOR GAMMA DISTRIBUTION)
      WRITE(6,435) (I,I=1,13)
  435 FORMAT(1H ,5HMONTH,5I9,8I10)
      WRITE(6,434) (AMBA(J),J=1,13)
  434 FORMAT(1H ,5HLAMBA,5F9.4,8F10.4)
      WRITE(6,436) (RPAR(I),I=1,13)
  436 FORMAT(1H ,5H  R  ,5F9.3,8F10.3)
      WRITE(6,437) (GAMMA(I),I=1,13)
  437 FORMAT(1H ,5HGAMMA,5E9.2,8E10.3)
      IF(NGAMMA .EQ. 1) GO TO 247
      WRITE(6,438)
  438 FORMAT(1H ,42HDENSITY FUNCTIONS FROM GAMMA DISTRIBUTIONS)
      WRITE(6,441)
  441 FORMAT(1H ,5HMONTH)
      DO 442 J=1,13
      WRITE(6,409) J,(XR(I,J),I=1,10)
  442 WRITE(6,419) J,(FXR(I,J),I=1,10)
  409 FORMAT(1H ,I5,14H RUNOFF(AC-FT),10F11.2)
  419 FORMAT(1H ,I5,14HDENSITY FUNCT.,10E11.4)
  247 IF(LNOR .EQ. 0 .OR. NN5 .EQ. 0) GO TO 50
      DO 31 J=1,13
      SUM=0.0
      DO 32 I=1,NN
      R(I,J)=ALOG(R(I,J)+1.0)
   32 SUM=SUM+R(I,J)
      AV(J)=SUM/FN
   31 CONTINUE
      DO 33 J=1,13
  330 SX2=0.0
```

```
      SX3=0.0
      AVE=AV(J)
      DO 34 I=1,NN
      SX=R(I,J)-AVE
      SXX=SX*SX
      SX2=SX2+SXX
   34 SX3=SX3+SX*SXX
      VAR=SX2/FN1
      STD(J)=SQRT(VAR)
      SKEW(J)=FN*SX3/(FN1*FN2*VAR*STD(J))
   33 CONTINUE
      WRITE(6,115) NB,NSTA
  115 FORMAT(1H0,72HTHE FOLLOWING IS OBTAINED ASSUMING A LOG-NORMAL DIST
     $RIBUTION FOR STATION,I3,1H-,I4)
      WRITE(6,134) (AV(J),J=1,13)
  134 FORMAT(1H ,4H AVE,4F9.4,9F10.4)
      WRITE(6,135) (STD(J),J=1,13)
      WRITE(6,136) (SKEW(J),J=1,13)
  135 FORMAT(1H ,4HSTD.,4F9.4,9F10.4)
  136 FORMAT(1H ,4HSKEW,4F9.5,9F10.5)
      WRITE(6,108) NB,NSTA
      WRITE(6,109)
      DO 35 I=1,NPH
      FAC=T(I,NN5)
      DO 36 J=1,13
  331 FACD=FAC*STD(J)
      RUN1(J)=EXP(AV(J)+FACD)
      RUN(J,I)=EXP(AV(J)-FACD)
   36 CONTINUE
      WRITE(6,111) P(I),(RUN1(J),J=1,13)
      DO 37 J=1,13
   37 RUN1(J)=CONV*RUN1(J)
   35 WRITE(6,112) (RUN1(J),J=1,13)
      DO 38 J=1,13
      RUN1(J)=EXP(AV(J))
   38 CONTINUE
      WRITE(6,111) P(NPH1),(RUN1(J),J=1,13)
      DO 39 J=1,13
   39 RUN1(J)=CONV*RUN1(J)
      WRITE(6,112) (RUN1(J),J=1,13)
      DO 40 I=1,NPH
      IM=NPH1-I
      II=NPH1+I
      WRITE(6,111) P(II),(RUN(J,IM),J=1,13)
      DO 41 J=1,13
   41 RUN(J,IM)=CONV*RUN(J,IM)
   40 WRITE(6,112) (RUN(J,IM),J=1,13)
      GO TO 50
   99 STOP
      END
 &IN FOR FINTE
      FUNCTION RLANGR(P0,P1,P2,P3,R1,R2,R3)
      A1=(P0-P2)*(P0-P3)*R1/((P1-P2)*(P1-P3))
      A2=(P0-P1)*(P0-P3)*R2/((P2-P1)*(P2-P3))
      A3=(P0-P1)*(P0-P2)*R3/((P3-P1)*(P3-P2))
      RLANGR=A1+A2+A3
      RETURN
      END
 &N XQT MONF
```

LISTING OF OUTPUT

ORIGINAL DATA FOR STATION 10- 230 (BIG CREEK NEAR RANDOLPH)

YEAR	OCT.	NOV.	DEC.	JAN.	FEB.	MARCH	APRIL	MAY	JUNE	JULY	AUG.	SEPT.	ANNUAL
1940	332.	360.	283.	307.	282.	301.	243.	136.	92.	63.	69.	95.	2563.
1941	154.	200.	92.	123.	194.	171.	151.	154.	79.	67.	133.	112.	1630.
1944	558.	519.	491.	461.	375.	470.	730.	938.	494.	388.	323.	344.	6091.
1950	944.	785.	676.	676.	555.	778.	2210.	5270.	3560.	2480.	1950.	1680.	21564.
1951	1520.	1350.	1230.	831.	817.	876.	2520.	4640.	2690.	2270.	1880.	1610.	22234.
1952	1620.	1350.	1090.	789.	692.	756.	1500.	5870.	3700.	2370.	1870.	1690.	23297.
1953	1600.	1260.	1140.	1130.	736.	930.	954.	1640.	1940.	1500.	1260.	1010.	15100.
1954	897.	774.	639.	585.	571.	702.	843.	1250.	976.	742.	559.	489.	9027.
1955	464.	429.	411.	390.	330.	395.	446.	1410.	960.	657.	542.	447.	6881.
1956	443.	436.	636.	567.	467.	843.	2010.	2920.	1980.	1550.	1220.	1010.	14082.
1957	966.	757.	660.	524.	589.	672.	714.	2440.	2020.	1570.	1290.	1080.	13282.
1958	964.	841.	768.	676.	605.	585.	746.	2360.	1690.	1130.	924.	781.	12070.
1959	734.	690.	675.	625.	617.	823.	1050.	950.	738.	619.	476.	497.	8494.
1960	477.	430.	430.	400.	387.	660.	584.	542.	420.	341.	320.	332.	5323.
1961	365.	350.	296.	214.	179.	339.	302.	231.	142.	91.	117.	143.	2769.
1962	187.	179.	184.	184.	509.	482.	2010.	2020.	831.	586.	559.	490.	8221.
1963	533.	486.	470.	399.	583.	384.	514.	735.	475.	375.	404.	381.	5739.
1964	361.	342.	307.	277.	262.	263.	484.	1050.	910.	527.	472.	451.	5706.
1965	446.	420.	641.	469.	425.	411.	1310.	4170.	2750.	1970.	1490.	1290.	15792.

BASED ON A NORMAL DISTRIBUTION

AV. STD. DEV. AND COEF. OF SKEW. FOR STATION 10- 230 (BIG CREEK NEAR RANDOLPH)

YEAR	OCT.	NOV.	DEC.	JAN.	FEB.	MARCH	APRIL	MAY	JUNE	JULY	AUG.	SEPT.	ANNUAL
AVE.	714.	629.	585.	507.	483.	571.	1017.	2038.	1392.	1016.	835.	733.	10519.
STD.	458.	362.	314.	250.	183.	231.	714.	1770.	1145.	817.	632.	531.	6724.
SKEW	.95	.97	.61	.68	-.07	-.06	.87	.99	.76	.60	.61	.70	.65

TABLE OF RUNOFF VALUES AT INDICATED PROBABILITY LEVEL FOR STATION 10- 230

PROB LEVEL	OCT.	NOV.	DEC.	JAN.	FEB.	MARCH	APRIL	MAY	JUNE	JULY	AUG.	SEPT.	ANNUAL
1.0	2063.	1697.	1511.	1243.	1021.	1252.	3120.	7255.	4766.	3423.	2697.	2299.	30335.
IN.	.741	.609	.543	.447	.367	.450	1.121	2.606	1.712	1.229	.969	.826	10.896
5.0	1690.	1401.	1254.	1039.	872.	1064.	2538.	5810.	3831.	2756.	2181.	1865.	24849.
IN.	.607	.503	.451	.373	.313	.382	.912	2.087	1.376	.990	.783	.670	8.925
10.0	1517.	1264.	1136.	945.	803.	976.	2268.	5141.	3399.	2447.	1942.	1665.	22307.
IN.	.545	.454	.408	.339	.288	.351	.815	1.847	1.221	.879	.698	.598	8.012
25.0	1030.	880.	802.	679.	609.	730.	1510.	3261.	2183.	1580.	1271.	1100.	15166.
IN.	.370	.316	.288	.244	.219	.262	.542	1.171	.784	.568	.457	.395	5.447
50.0	714.	629.	585.	507.	483.	571.	1017.	2038.	1392.	1016.	835.	733.	10519.
IN.	.256	.226	.210	.182	.173	.205	.365	.732	.500	.365	.300	.263	3.778
75.0	398.	379.	368.	334.	357.	411.	524.	815.	601.	451.	398.	366.	5873.
IN.	.143	.136	.132	.120	.128	.148	.188	.293	.216	.162	.143	.132	2.109
90.0	-89.	-5.	35.	69.	163.	165.	-234.	-1065.	-615.	-416.	-273.	-198.	-1268.
IN.	-.032	-.002	.012	.025	.058	.059	-.084	-.382	-.221	-.150	-.098	-.071	-.456
95.0	-262.	-142.	-84.	-26.	94.	78.	-504.	-1734.	-1048.	-725.	-512.	-399.	-3810.
IN.	-.094	-.051	-.030	-.009	.034	.028	-.181	-.623	-.376	-.260	-.184	-.143	-1.369
99.0	-635.	-438.	-340.	-230.	-55.	-111.	-1086.	-3178.	-1982.	-1392.	-1027.	-832.	-9297.
IN.	-.228	-.157	-.122	-.083	-.020	-.040	-.390	-1.142	-.712	-.500	-.369	-.299	-3.339

PROBABILITIES BASED ON RANKING

RANKED VALUES OF RUNOFF FOR STATION 10- 230 (BIG CREEK NEAR RANDOLPH)

PROB LEVEL	OCT.	NOV.	DEC.	JAN.	FEB.	MARCH	APRIL	MAY	JUNE	JULY	AUG.	SEPT.	ANNUAL
5.0	1620.	1350.	1230.	1130.	817.	930.	2520.	5870.	3700.	2480.	1950.	1690.	23297.
10.0	1600.	1350.	1140.	831.	736.	876.	2210.	5270.	3560.	2370.	1880.	1680.	22234.
15.0	1520.	1260.	1090.	789.	692.	843.	2010.	4640.	2750.	2270.	1870.	1610.	21564.
20.0	966.	841.	768.	676.	617.	823.	2010.	4170.	2690.	1970.	1490.	1290.	15792.
25.0	964.	785.	676.	676.	605.	778.	1500.	2920.	2020.	1570.	1290.	1080.	15100.
30.0	944.	774.	675.	625.	589.	756.	1310.	2440.	1980.	1550.	1260.	1010.	14082.
35.0	897.	757.	660.	585.	583.	702.	1050.	2360.	1940.	1500.	1220.	1010.	13282.
40.0	734.	690.	641.	567.	571.	672.	954.	2020.	1690.	1130.	924.	781.	12070.
45.0	558.	519.	639.	524.	555.	660.	843.	1640.	976.	742.	559.	497.	9027.
50.0	533.	486.	636.	469.	509.	585.	746.	1410.	960.	657.	559.	490.	8494.
55.0	477.	436.	491.	461.	467.	482.	730.	1250.	910.	619.	542.	489.	8221.
60.0	464.	430.	470.	400.	425.	470.	714.	1050.	831.	586.	476.	451.	6881.
65.0	446.	429.	430.	399.	387.	411.	584.	950.	738.	527.	472.	447.	6091.
70.0	443.	420.	411.	390.	375.	395.	514.	938.	494.	388.	404.	381.	5739.
75.0	365.	360.	307.	307.	330.	384.	484.	735.	475.	375.	323.	344.	5706.
80.0	361.	350.	296.	277.	282.	339.	446.	542.	420.	341.	320.	332.	5323.
85.0	332.	342.	283.	214.	262.	301.	302.	231.	142.	91.	133.	143.	2769.
90.0	187.	200.	184.	184.	194.	263.	243.	154.	92.	67.	117.	112.	2563.
95.0	154.	179.	92.	123.	179.	171.	151.	136.	79.	63.	69.	95.	1630.

TABLE OF RUNOFF VALUES AT INDICATED PROBABILITY LEVEL FOR STATION 10- 230

PROB LEVEL	OCT.	NOV.	DEC.	JAN.	FEB.	MARCH	APRIL	MAY	JUNE	JULY	AUG.	SEPT.	ANNUAL
1.0	1593.	1285.	1331.	1554.	908.	988.	2847.	6328.	3330.	2575.	2049.	1655.	24430.
IN.	.572	.462	.478	.558	.326	.355	1.023	2.273	1.196	.925	.736	.594	8.775
5.0	1620.	1350.	1230.	1130.	817.	930.	2520.	5870.	3700.	2480.	1950.	1690.	23297.
IN.	.582	.485	.442	.406	.293	.334	.905	2.108	1.329	.891	.700	.607	8.368
10.0	1600.	1350.	1140.	831.	736.	876.	2210.	5270.	3560.	2370.	1880.	1680.	22234.
IN.	.575	.485	.409	.298	.264	.315	.794	1.893	1.279	.851	.675	.603	7.986
25.0	964.	785.	676.	676.	605.	778.	1500.	2920.	2020.	1570.	1290.	1080.	15100.
IN.	.346	.282	.243	.243	.217	.279	.539	1.049	.726	.564	.463	.388	5.424
50.0	533.	486.	636.	469.	509.	585.	746.	1410.	960.	657.	559.	490.	8494.
IN.	.191	.175	.228	.168	.183	.210	.268	.506	.345	.236	.201	.176	3.051
75.0	365.	360.	307.	307.	330.	384.	484.	735.	475.	375.	323.	344.	5706.
IN.	.131	.129	.110	.110	.119	.138	.174	.264	.171	.135	.116	.124	2.050
90.0	187.	200.	184.	184.	194.	263.	243.	154.	92.	67.	117.	112.	2563.
IN.	.067	.072	.066	.066	.070	.094	.087	.055	.033	.024	.042	.040	.921
95.0	154.	179.	92.	123.	179.	171.	151.	136.	79.	63.	69.	95.	1630.
IN.	.055	.064	.033	.044	.064	.061	.054	.049	.028	.023	.025	.034	.585
99.0	208.	249.	23.	52.	205.	59.	54.	164.	95.	74.	8.	91.	360.
IN.	.075	.090	.008	.019	.074	.021	.019	.059	.034	.027	.003	.033	.129

STATION 10- 230 NAME OF STATION (BIG CREEK NEAR RANDOLPH)

RUNOFF IN AC-FT AND INCHES FOR GIVEN PROBABILITY LEVELS USING A GAMMA DISTRIBUTION

PROB.	1.00	5.00	10.00	25.00	50.00	75.00	90.00	95.00	99.00
MONTH									
1	126.	229.	246.	393.	614.	948.	1364.	1672.	2047.
	.045	.082	.088	.141	.220	.341	.490	.600	.735
2	159.	246.	257.	377.	552.	816.	1145.	1387.	1684.
	.057	.088	.092	.135	.198	.293	.411	.498	.605
3	164.	179.	253.	357.	530.	757.	1015.	1197.	1429.
	.059	.064	.091	.128	.190	.272	.365	.430	.513
4	163.	336.	237.	324.	462.	640.	847.	1000.	1265.
	.059	.121	.085	.117	.166	.230	.304	.359	.454
5	279.	247.	295.	352.	451.	581.	728.	828.	936.
	.100	.089	.106	.126	.162	.209	.261	.297	.336
6	388.	280.	310.	401.	532.	699.	884.	993.	1101.
	.140	.101	.111	.144	.191	.251	.318	.357	.395
7	107.	262.	294.	526.	868.	1395.	2071.	2601.	3282.
	.038	.094	.106	.189	.312	.501	.744	.934	1.179
8	64.	293.	517.	809.	1625.	2934.	4686.	6147.	8090.
	.023	.105	.186	.291	.584	1.054	1.683	2.208	2.906
9	57.	251.	414.	601.	1127.	1973.	3077.	3960.	5025.
	.021	.090	.149	.216	.405	.709	1.105	1.422	1.805
10	45.	192.	213.	448.	829.	1431.	2203.	2763.	3377.
	.016	.069	.076	.161	.298	.514	.791	.992	1.213
11	42.	176.	197.	381.	696.	1159.	1734.	2142.	2583.
	.015	.063	.071	.137	.250	.416	.623	.769	.928
12	61.	170.	195.	360.	616.	1005.	1486.	1832.	2212.
	.022	.061	.070	.129	.221	.361	.534	.658	.795
13	1413.	3481.	3672.	5808.	9337.	14382.	20703.	25466.	30752.
	.507	1.250	1.319	2.086	3.354	5.166	7.436	9.147	11.046

PARAMETERS FOR GAMMA DISTRIBUTION

MONTH	1	2	3	4	5	6	7	8	9	10	11	12	13
LAMBA	.0030	.0038	.0055	.0068	.0101	.0083	.0019	.0007	.0011	.0015	.0021	.0025	.0002
R	1.827	1.889	3.013	2.879	3.384	3.645	1.798	1.326	1.479	1.546	1.745	1.763	2.135
GAMMA	.94-00	.96-00	.20+01	.18+01	.29+01	.391+01	.931-00	.894-00	.886-00	.889-00	.918-00	.922-00	.106+01

THE FOLLOWING IS OBTAINED ASSUMING A LOG-NORMAL DISTRIBUTION FOR STATION 10- 230

AVE	6.3733	6.2926	6.2090	6.0996	6.1009	6.2547	6.6693	7.1547	6.7687	6.4608	6.3658	6.2932	9.0294
STD.	.6670	.5776	.6427	.5555	.4365	.4706	.7750	1.1245	1.1636	1.1582	.9773	.8748	.7529
SKEW	-.18750	-.06569	-.95753	-.67274	-.76599	-.74151	-.34526	-.65904	-.81513	-.83291	-.65784	-.53963	-.57156

TABLE OF RUNOFF VALUES AT INDICATED PROBABILITY LEVEL FOR STATION 10- 230

PROB LEVEL	OCT.	NOV.	DEC.	JAN.	FEB.	MARCH	APRIL	MAY	JUNE	JULY	AUG.	SEPT.	ANNUAL
1.0	4183.	2965.	3305.	2291.	1615.	2083.	7733.	35198.	26848.	19416.	10362.	7124.	76744.
IN.	1.503	1.065	1.187	.823	.580	.748	2.778	12.643	9.644	6.974	3.722	2.559	27.566
5.0	2427.	1851.	1956.	1456.	1131.	1419.	4109.	14060.	10388.	7546.	4668.	3489.	41517.
IN.	.872	.665	.703	.523	.406	.510	1.476	5.050	3.731	2.711	1.677	1.253	14.913
10.0	1886.	1488.	1534.	1180.	959.	1188.	3065.	9191.	6691.	4871.	3226.	2507.	31234.
IN.	.678	.534	.551	.424	.345	.427	1.101	3.302	2.403	1.750	1.159	.900	11.219
[illegible]	929.	806.	775.	654.	603.	720.	1346.	2784.	1944.	1424.	1143.	990.	14040.
IN.	.334	[illegible]	[illegible]	.235	.217	.259	.483	1.000	.698	.511	.410	.356	5.043
50.0	[illegible]	541.	497.	[illegible]	[illegible]	[illegible]	788.	1280.	870.	640.	582.	541.	8345.
IN.	.210	.194	.179	.160	.160	.187	.283	.460	.313	.230	.209	.194	2.997
75.0	370.	363.	319.	304.	330.	376.	461.	589.	389.	287.	296.	296.	4960.
IN.	.133	.130	.115	.109	.119	.135	.166	.211	.140	.103	.106	.106	1.782
90.0	182.	196.	161.	168.	208.	228.	202.	178.	113.	84.	105.	117.	2230.
IN.	.065	.071	.058	.060	.075	.082	.073	.064	.041	.030	.038	.042	.801
95.0	141.	158.	126.	136.	176.	191.	151.	117.	73.	54.	72.	84.	1677.
IN.	.051	.057	.045	.049	.063	.069	.054	.042	.026	.019	.026	.030	.602
99.0	82.	99.	75.	87.	123.	130.	80.	47.	28.	21.	33.	41.	907.
IN.	.029	.035	.027	.031	.044	.047	.029	.017	.010	.008	.012	.015	.326